

Chiral Phase Transition beyond Mean Field Approximation

Lianyi He and Pengfei Zhuang

Physics Department, Tsinghua University, Beijing 100084, China

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Based on the analogy between the Nambu–Jona-Lasinio model of chiral symmetry breaking and the BCS theory of superconductivity, we investigate the effect of $\bar{q}q$ pair fluctuations on the chiral phase transition. We include uncondensed $\bar{q}q$ pairs at finite temperature and chemical potential in a self-consistent T-matrix formalism, the so-called G_0G scheme. The pair fluctuations reduce significantly the critical temperature and make quarks massive above the critical temperature.

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It is well known that the spontaneous breaking of chiral symmetry is one of the most important features of the vacuum of Quantum Chromodynamics (QCD). The spontaneous breaking successfully explains many low energy phenomena in QCD, such as the large continuum quark mass and the small pion mass. The quark-antiquark condensate $\langle \bar{q}q \rangle$ which characterizes the spontaneous breaking is about $(250 \text{ MeV})^3$ from the QCD sum rule and lattice calculations. It is generally believed that, in hot and dense medium, the chiral condensate will be reduced and the broken chiral symmetry will be restored at sufficiently high temperature and density.

Since the chiral symmetry breaking and restoration happen in the non-perturbative region, it is hard to handle them directly by the original QCD lagrangian. The investigation relies mainly on low energy effective models and lattice calculations. One of the successful models is the Nambu–Jona-Lasinio (NJL) model [1, 2]. The idea of the model is from the well-known Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. In superconducting metals, the Cooper pairing between electrons in the spin-singlet state leads to a condensate $\langle \psi \uparrow \psi \downarrow \rangle$ which spontaneously breaks the electromagnetic U(1) symmetry and causes an energy gap. In analogy, the pairing between a quark and an antiquark with the same chirality leads to a condensate $\langle \bar{q}q \rangle$. Such a condensate spontaneously breaks the chiral symmetry and gives a dynamical mass to the quarks.

While the analogy between the QCD and BCS vacua leads to a successful theory of chiral symmetry breaking, one should keep in mind that there is an important difference between the two: In contrast to the BCS superconductivity, the chiral symmetry breaking is a strong coupling phenomenon. From the recent studies on relativistic heavy ion collisions, there may exist a strongly coupled quark-gluon plasma phase (sQGP) [3] above the critical temperature T_c of the deconfinement and chiral phase transitions where both quarks and their bound states are constituents of the system. From lattice simulations, any thermodynamic quantity can not reach its Stefan-Boltzmann limit even at extremely high temperature [4]. A possible explanation for this thermodynamic suppression is that quarks have a large thermal mass in the deconfined and chiral restored phase [3, 5, 6].

In the language of condensed matter physics, these

phenomena strongly indicate that there exists a pseudogap for quarks [7, 8, 9, 10] and the matter may be in the crossover region from BCS to Bose-Einstein condensation (BEC) [11, 12, 13]. It is generally believed that the BCS mean field theory describes well the crossover at zero temperature. However, to obtain a quantitatively correct T_c and a correct fermion excitation above T_c , one should properly consider pair fluctuations at finite temperature. There are some T-matrix schemes to treat the finite temperature pair fluctuations. Among them, the self-consistent pair approximation or the so-called G_0G scheme [13, 14, 15] is a competitive one. The effect of the pair fluctuations in this scheme is treated as a pseudogap. Recently, the G_0G theory is applied to the study of color superconductivity in the NJL model [16]. In this paper, we will investigate the chiral phase transition in the NJL model, by using the self-consistent G_0G theory.

To have a BCS-like description of the spontaneous chiral symmetry breaking, it is useful to take the two-dimension Weyl spinors defined as

$$q = \begin{pmatrix} q_L \\ q_R \end{pmatrix}, \quad \bar{q} = \begin{pmatrix} q_R^\dagger & q_L^\dagger \end{pmatrix} \quad (1)$$

where the color and flavor indexes of the quark field are not explicitly shown. The kinematic term of the NJL Lagrangian, $\mathcal{L}_0 = \bar{q}i\not{D}q$, can be written as

$$\mathcal{L}_0 = q_L^\dagger i(\partial_t - \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})q_L + q_R^\dagger i(\partial_t + \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})q_R, \quad (2)$$

where σ_i are the Pauli matrices. The interaction between quarks and antiquarks in the NJL model are expressed by a four fermion coupling term, it can be generally written as

$$\mathcal{L}_I = \frac{g}{4} \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\Gamma q)^2 \right], \quad (3)$$

where $\Gamma = 1$ and $\boldsymbol{\sigma}$ are, respectively, for one and two flavor cases. To see directly the analogy between the NJL model of chiral symmetry breaking and the BCS theory of superconductivity, it is intuitive to look into the one flavor case. Using the identities for the Weyl spinors,

$$\begin{aligned} \bar{q}q &= q_R^\dagger q_L + q_L^\dagger q_R \\ \bar{q}i\gamma_5 q &= -i(q_R^\dagger q_L - q_L^\dagger q_R), \end{aligned} \quad (4)$$

the four fermion interaction can be written as

$$\mathcal{L}_I = g q_R^\dagger q_L q_L^\dagger q_R, \quad (5)$$

which has the same structure of the interaction between fermions in BCS theory where one has $\mathcal{L}_I = g \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow$.

The order parameter field describing chiral symmetry breaking can be defined as $\Phi(t, \mathbf{x}) = -g q_R^\dagger q_L$. The QCD vacuum is characterized by the chiral condensate $\langle \Phi \rangle$ which means the pairing between a quark and an antiquark with the same chirality. Since the vacuum has a definite parity, the chiral condensate is a real number, $\langle q_R^\dagger q_L \rangle = \langle q_L^\dagger q_R \rangle$.

We start by rewriting the mean field theory of the two flavor NJL model in a T-matrix formalism. Such a formalism is important for us to go beyond the mean field by including the non-condensed $\bar{q}q$ pairs at finite temperature. The mean field quark propagator \mathcal{S} reads

$$\mathcal{S}^{-1}(k) = \begin{pmatrix} \mathcal{G}_{0L}^{-1}(k) & -m_{sc} \\ -m_{sc} & \mathcal{G}_{0R}^{-1}(k) \end{pmatrix}, \quad (6)$$

where $\mathcal{G}_{0L,R}^{-1}(k) = i\nu_n + \mu \pm \boldsymbol{\sigma} \cdot \mathbf{k}$ are the free quark propagators with ν_n being the fermion frequency $\nu_n = (2n+1)\pi T$ ($n = 0, \pm 1, \pm 2, \dots$), and $m_{sc} = \langle \Phi \rangle$ is the order parameter of the phase transition. The quark chemical potential μ is introduced by considering conserved charge density $q^\dagger q$. The explicit form of the quark propagator can be expressed as

$$\mathcal{S}(k) = \begin{pmatrix} \mathcal{G}_L(k) & \mathcal{F}_L(k) \\ \mathcal{F}_R(k) & \mathcal{G}_R(k) \end{pmatrix} \quad (7)$$

with the matrix elements given by

$$\begin{aligned} \mathcal{G}_{L,R}(k) &= [\mathcal{G}_{0L,R}^{-1}(k) - \Sigma_{L,R}(k)]^{-1}, \\ \mathcal{F}_{L,R}(k) &= m_{sc} \mathcal{G}_{L,R}(k) \mathcal{G}_{0R,L}(k), \end{aligned} \quad (8)$$

where the self-energies $\Sigma_{L,R}$ at mean field level are defined as

$$\Sigma_{L,R}(k) = m_{sc}^2 \mathcal{G}_{0R,L}(k). \quad (9)$$

After some simple algebra, the diagonal and off-diagonal elements can be evaluated as

$$\begin{aligned} \mathcal{G}_{L,R}(k) &= \frac{i\nu_n + \mu \mp \boldsymbol{\sigma} \cdot \mathbf{k}}{(i\nu_n + \mu)^2 - \mathbf{k}^2 - m_{sc}^2}, \\ \mathcal{F}_{L,R}(k) &= \frac{m_{sc}}{(i\nu_n + \mu)^2 - \mathbf{k}^2 - m_{sc}^2}. \end{aligned} \quad (10)$$

For those who are familiar with the BCS theory of superconductivity, \mathcal{G} and \mathcal{F} are in analogy to the normal and anomalous Green functions, and the quark chemical potential plays the role of effective Zeeman splitting between quarks and antiquarks. At $\mu = 0$, the quark propagator has two poles $i\nu_n = \pm \sqrt{\mathbf{k}^2 + m_{sc}^2}$, which means that quarks obtain a mass gap m_{sc} in the chiral symmetry

breaking phase. The order parameter m_{sc} is determined by the self-consistent gap equation

$$m_{sc} = -g \sum_k \text{Tr} \mathcal{F}_{L,R}(k), \quad (11)$$

where the summation over quark momentum is defined as $\sum_k = T \sum_n \int d^3\mathbf{k} / (2\pi)^3$ and the trace is taken in color and flavor spaces.

In the mean field theory, $\bar{q}q$ pairs enter into the problem only through the condensate. The condensed pairs can be associated with a T-matrix in such a way

$$t_{sc}(q) = \frac{m_{sc}^2}{T} \delta(q) \quad (12)$$

that the quark self energies (9) can be formally expressed as

$$\Sigma_{L,R}(k) = \sum_q t_{sc}(q) \mathcal{G}_{0R,L}(k - q). \quad (13)$$

From the gap equation (11), the mean field theory is related to a particular asymmetric pair susceptibility [13],

$$\begin{aligned} \chi(q) &= \chi_{LR}(q) = \chi_{RL}(q) \\ &= \frac{1}{2} \sum_k \text{Tr} \left[\mathcal{G}_L(k) \mathcal{G}_{0R}(k - q) \right. \\ &\quad \left. + \mathcal{G}_L(k - q) \mathcal{G}_{0R}(k) \right], \end{aligned} \quad (14)$$

from which the gap equation in the symmetry breaking phase is given by the condition

$$1 + g\chi(0) = 0. \quad (15)$$

This suggests that the uncondensed $\bar{q}q$ pair propagator should be in the form

$$t_{pg}(q) = \frac{g}{1 + g\chi(q)}. \quad (16)$$

The gap equation (15) is just the so-called BEC condition $t_{pg}^{-1}(q = 0) = 0$. While the uncondensed $\bar{q}q$ pairs play no role in the BCS mean field theory, such a specific choice of the pair susceptibility and the BEC condition are the fundamental criterion for us to go beyond the mean field.

We now take into account the uncondensed $\bar{q}q$ pairs. In the mean field theory, the quark self-energies $\Sigma_{L,R}$ contain only the contribution from the condensed $\bar{q}q$ pairs. This is mostly correct at zero temperature. However, at finite temperature, the condensed pairs with zero total momentum can be thermally excited. Therefore, the total contribution to the quark self-energies should include both the condensed (sc) pairs and the uncondensed or “pseudogap”-associated (pg) pairs[13],

$$\Sigma_{L,R}(k) = \sum_q t(q) \mathcal{G}_{0R,L}(k - q) \quad (17)$$

with the total pair propagator $t(q)$ defined by

$$\begin{aligned} t(q) &= t_{\text{pg}}(q) + t_{\text{sc}}(q), \\ t_{\text{pg}}(q) &= \frac{g}{1 + g\chi(q)}, \quad q \neq 0, \\ t_{\text{sc}}(q) &= \frac{m_{\text{sc}}^2}{T} \delta(q). \end{aligned} \quad (18)$$

With the modified self-energies, the dressed quark propagators $\mathcal{G}_{\text{L,R}}(k)$ and the pair susceptibility $\chi(q)$ are still given by (8) and (14). The beyond-BCS effect is reflected in the quark self-energies $\Sigma_{\text{L,R}}$. The BEC condition $t_{\text{pg}}^{-1}(0) = 0$ and the quark self-energies form in principle a coupled set of equations to determine the new order parameter m_{sc} .

The above equations are hard to handle analytically. In the symmetry breaking phase with $T \leq T_c$, the BEC condition $t_{\text{pg}}^{-1}(0) = 0$ implies that $t_{\text{pg}}(q)$ is strongly peaked at $q = 0$. This allows us to approximate[13]

$$\Sigma_{\text{L,R}}(k) \simeq m^2 \mathcal{G}_{0\text{R,L}}(k), \quad (19)$$

where m^2 includes the contribution from both the condensed and the thermally excited $\bar{q}q$ pairs,

$$m^2 = m_{\text{sc}}^2 + m_{\text{pg}}^2, \quad (20)$$

with the pseudogap m_{pg} defined as

$$m_{\text{pg}}^2 = \sum_{q \neq 0} t_{\text{pg}}(q). \quad (21)$$

Note that, above the critical temperature T_c , the BEC condition disappears and then such an approximation is in principle not valid. How to practically treat the pair fluctuations and their effect on the quark propagator above T_c is still an open question in the G_0G scheme. However, for temperatures not much higher than T_c , this approximation may be still good to give qualitatively correct result.

Under the approximation, the dressed quark propagator can be analytically evaluated as

$$\mathcal{G}_{\text{L,R}}(k) = \frac{i\nu_n + \mu \mp \boldsymbol{\sigma} \cdot \mathbf{k}}{(i\nu_n + \mu)^2 - \mathbf{k}^2 - m^2}, \quad (22)$$

which means that, when one goes beyond the mean field, quarks obtain a mass gap $m = (m_{\text{sc}}^2 + m_{\text{pg}}^2)^{1/2}$ rather than m_{sc} . One may worry about that the appearance of the pseudogap m_{pg} explicitly breaks the chiral symmetry. Parallel to the discussion in the BCS-BEC crossover[15], we can show that m_{pg}^2 is just the classical fluctuation of the order parameter field Φ ,

$$m_{\text{pg}}^2 = \langle |\Phi|^2 \rangle - \langle |\Phi| \rangle^2, \quad (23)$$

and hence does not break the chiral symmetry.

After introducing the pseudogap m_{pg} , the pair susceptibility can be analytically evaluated as

$$\begin{aligned} \chi(q) &= N_c N_f \sum_{e=\pm} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{|\mathbf{k} - \mathbf{q}| + eE_{\mathbf{k}}}{(i\omega_n)^2 - (|\mathbf{k} - \mathbf{q}| + eE_{\mathbf{k}})^2} \\ &\quad \left[1 + e \frac{|\mathbf{k}|}{E_{\mathbf{k}}} \cos \phi_{\mathbf{q}} \right] \left[1 - \bar{f}(eE_{\mathbf{k}}) - \bar{f}(|\mathbf{k} - \mathbf{q}|) \right], \end{aligned} \quad (24)$$

where N_c and N_f are color and flavor numbers, and the pair frequency ω_n , quark energy $E_{\mathbf{k}}$, the angle $\phi_{\mathbf{q}}$ and the function $\bar{f}(x)$ are defined as $\omega_n = 2n\pi T$ ($n = 0, \pm 1, \pm 2, \dots$), $E_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$, $\cos \phi_{\mathbf{q}} = \mathbf{k} \cdot (\mathbf{k} - \mathbf{q}) / (|\mathbf{k}| |\mathbf{k} - \mathbf{q}|)$ and $\bar{f}(x) = [f(x - \mu) + f(x + \mu)] / 2$ with $f(x) = 1 / (e^{x/T} + 1)$ being the Fermi-Dirac distribution function. The BEC condition $1 + g\chi(0) = 0$, namely the gap equation can now be explicitly written as

$$1 - gN_c N_f \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1 - 2\bar{f}(E_{\mathbf{k}})}{E_{\mathbf{k}}} = 0. \quad (25)$$

The equations (21) and (25) form a closed set to determine the order parameter m_{sc} and the pseudogap m_{pg} as functions of temperature and chemical potential.

Solving such a coupled set of equations is still rather complicated. In the temperature region below and around the critical temperature T_c which is expected to be much smaller than the momentum cutoff Λ of the NJL model, we can employ the pole approximation for the pair propagator t_{pg} ,

$$t_{\text{pg}}(\omega, \mathbf{q}) \simeq -\frac{Z^{-1}}{\omega^2 - \omega_{\mathbf{q}}^2} \quad (26)$$

with $\omega_{\mathbf{q}}^2 = v^2 \mathbf{q}^2 + \Delta^2$, where we have taken the analytical continuation $i\omega_n \rightarrow \omega + i0^+$, and the coefficients Z, v^2 and Δ^2 are defined as

$$\begin{aligned} Z &= -\frac{1}{2} \frac{\partial^2 \chi}{\partial \omega^2} \Big|_{\omega=\mathbf{q}=0}, \quad v^2 = \frac{1}{2Z} \frac{\partial^2 \chi}{\partial \mathbf{q}^2} \Big|_{\omega=\mathbf{q}=0} \\ \Delta^2 &= \frac{1}{gZ} [1 + g\chi(0)]. \end{aligned} \quad (27)$$

The pair width is safely neglected below and around T_c [13]. With the help of the pole approximation, the pseudogap equation (21) takes the simple form

$$m_{\text{pg}}^2 = \frac{1}{Z} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{1 + 2b(\omega_{\mathbf{q}})}{2\omega_{\mathbf{q}}}, \quad (28)$$

where $b(x) = 1 / (e^{x/T} - 1)$ is the Bose distribution function. Since the vacuum is well described by the mean field theory, we require $m_{\text{pg}} = 0$ at zero temperature and chemical potential. To this end, we simply eliminate the first term on the righthand side of (28).

Before numerical calculations, we can analytically arrive at the following conclusions:

1) In the symmetry breaking phase, the gap equation

$1 + g\chi(0) = 0$ leads to $\Delta^2 = 0$, and then the pair dispersion is gapless in the long wavelength limit. At sufficiently low temperature, the pseudogap m_{pg} is proportional to the temperature T .

2) From $\chi, Z \sim N_c$, we have $m_{pg}^2 \sim 1/N_c$. In the large N_c limit, the pseudogap effect can be neglected, and the mean field approximation describes well the chiral phase transition.

The NJL model is non-renormalizable, the simplest regularization scheme is to use a three momentum cutoff Λ to regularize the integrals over quark and pair momenta. In the vacuum, the spontaneous breaking of chiral symmetry occurs when the coupling exceeds the critical value $g_c = 4\pi^2/(N_c N_f \Lambda^2)$. In the following we use a dimensionless quantity $\eta = g/g_c - 1$ to denote the coupling strength. To fit the pion decay constant $f_\pi = 94$ MeV and constituent quark mass $m = 312$ MeV in the vacuum, we take $\Lambda = 653$ MeV and $g/4 = 5.01$ (GeV) $^{-2}$ [17] which result in the physical coupling $\eta = 0.3$. In Fig.1 we calculate the transition temperature T_c as a function of the coupling in mean field approximation and in the case including pseudogap effect with $N_c = 3$ and 100. It is clear that the pseudogap effect can be neglected at large enough N_c and disappears in the limit $N_c \rightarrow \infty$. For a finite value of N_c , the transition temperature with contribution from pair fluctuations is lower than the one in mean field approximation, and the difference between the two increases with increasing coupling. This conclusion is consistent with the results obtained in other approaches, like the non-linear sigma model approach of the NJL model [7, 8].

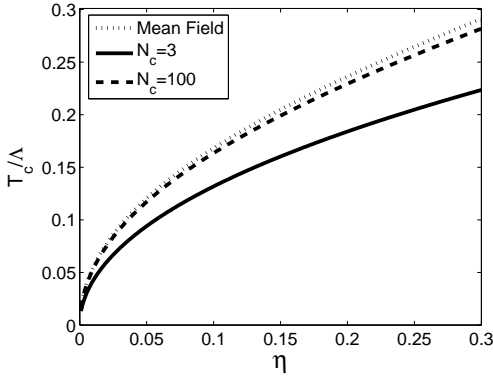


FIG. 1: The critical temperature T_c of chiral phase transition as a function of the coupling strength η in mean field approximation (dotted line) and in the case including pseudogap effect with $N_c = 3$ (solid line) and 100 (dashed line). The chemical potential μ is taken to be zero.

In the following we focus on the physical case of $N_c = 3$ and $\eta = 0.3$. In Fig.2 we show the temperature dependence of the order parameter m_{sc} , the pseudogap m_{pg} and the total quark mass m at zero chemical potential. At zero temperature, the pseudogap is zero and the quark mass comes purely from the dynamical symmetry break-

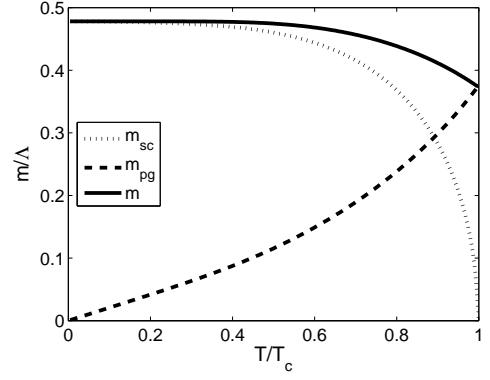


FIG. 2: The temperature dependence of the order parameter m_{sc} (dotted line), the pseudogap m_{pg} (dashed line) and the total quark mass m (solid line) at $\mu = 0$, $\eta = 0.3$ and $N_c = 3$.

ing. In the low temperature domain $T < 0.4 T_c$, there is a good linear relation between m_{pg} and T . With increasing temperature, while the order parameter is reduced, the pseudogap goes up and dominates the quark mass near the transition temperature. Due to the large pseudogap around the transition, the effective quark mass at $T_c = 146$ MeV is about 80% of its value in the vacuum. This is in contrary to the mean field picture where the quark mass is equivalently considered as the order parameter of chiral phase transition and then keeps zero in the chiral symmetry restoration phase.

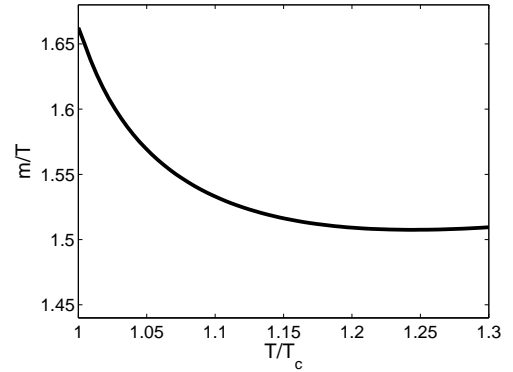


FIG. 3: The scaled quark mass m/T as a function of scaled temperature T/T_c at $\mu = 0$, $\eta = 0.3$ and $N_c = 3$ in the region above but close to the critical temperature T_c .

Above the transition temperature T_c , the order parameter m_{sc} vanishes, but the pseudogap m_{pg} is generally not zero in the temperature region close to T_c . As we mentioned above, the approximation (19) is in principle not valid above the transition temperature. However, for temperatures not much larger than T_c , it can be used to predict the quantitative behavior of pair fluctuations. In Fig.3, we calculate the quark mass m in the tempera-

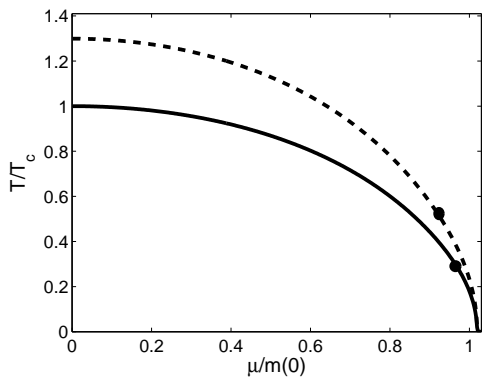


FIG. 4: The phase diagram in the $T - \mu$ plane with $\eta = 0.3$ and $N_c = 3$. The solid and dashed lines are, respectively, the phase transition lines with and without considering pair fluctuations. The black dots indicate the corresponding tricritical points.

ture domain $T_c < T < T_c^{\text{mf}} = 1.3T_c$, where $T_c^{\text{mf}} = 190$ MeV is the critical temperature at $\mu = 0$ in mean field approximation. In this temperature region, the quark mass comes purely from the pseudogap. For temperatures $T > 1.2T_c$, there is approximately a linear relation between the quark mass and temperature, $m \propto T$. This pseudogap induced quark mass can explain the fact why any thermodynamic function can not reach its Stefan-Boltzmann limit found by lattice calculations [4].

We now turn to the case of finite chemical potential. The quark chemical potential leads to a mismatch between the quark and antiquark Fermi surfaces, like the magnetic field which results in a Zeeman splitting between the spin-up and spin-down electrons in a superconductor. The phase diagram in the $T - \mu$ plane is shown

in Fig.4 where $T_c = 146$ MeV is the critical temperature at $\mu = 0$ and $m(0) = 312$ MeV is the quark mass in the vacuum. The critical temperature is reduced by pair fluctuations, but the pseudogap effect on the phase transition decreases with increasing chemical potential. At zero temperature, the two phase transition lines with and without considering pair fluctuations coincide. In both cases, the phase transition is of second order at low chemical potential and first order at high chemical potential. The tricritical point is shifted from $(T, \mu) = (78, 285)$ MeV in mean field approximation to $(45, 300)$ MeV when pair fluctuations are taken into account. In between the two phase transition lines, there exist quarks and pairs, like a superconductor in a magnetic field [18].

It is necessary to note that, the pairs considered in the G_0G scheme are not the real collective excitation modes [13], they just reflect the beyond-BCS effect on the fermion propagator. The real collective modes, especially the Goldstone modes, are constructed by the dressed fermion propagators $G_{L,R}$ [13].

In summary, based on the analogy between the mechanisms of chiral symmetry breaking and superconducting, we have extended the BCS-BEC crossover theory in the NJL model to including $\bar{q}q$ pair fluctuations at finite temperature and density. By using the BEC condition in the symmetry breaking phase, the fluctuations are treated as a quark pseudogap which reduces the critical temperature of chiral phase transition. While the NJL model lacks confinement and gluon degrees of freedom, the $\bar{q}q$ pair fluctuations induce a large pseudogap above the critical temperature which can be used to explain the large thermal quark mass observed in lattice QCD.

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